Connecting Polarization Observables and Amplitudes in Meson Photo-production

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- goals of *complete* experiments
- relations between observables defined in different conventions
- pseudoscalar meson photo-production cross sections
  with single, double and triple polarization
- constraints from recoil polarization
- connection between observables and CGLN F(*i*)
- Fierz identities connecting observables

Goals of *complete* experiments

I. extract the amplitudes:

- high-precision extraction (minimal or no model dependence) from measurements of pseudoscalar meson photo-production cross sections and polarization asymmetries

⇒ $A_I(W)$, an energy-dependent curve in the complex plane
→ eg. talk by Sam Hoblit (this session)

II. interpret the amplitudes:

eg. EBAC/Hall-B Joint Analysis project
→ talks by Toru Sato (this AM), Harry Lee (Wed AM)

- use extracted $A_I(W)$ amplitudes as a starting point for an analytic continuation into the complex plane to search for poles

⇒ evolution of the amplitude in the complex plane provides connection between "bare states" and physical resonances with coupled-channel dressings of the strong vertex

- eg. evolution of the double-Roper → session II-B

I. Amplitude extraction - overview:

- key channels in the resonance region: πN, $\pi\pi N$, $K\Lambda$, $K\Sigma$

  new information

  possibility for *complete* exps

- avoiding ambiguities will require asymmetries involving recoil polarization
  $\Rightarrow$ practical limitation to exps with simultaneous recoil polarization analysis:
  - $\gamma p \to K^+\Lambda$, $\gamma n \to K^0\Lambda$, using efficient self-analyzing weak decays
  - possibly $\gamma N \to \pi N$, using large cross sections

- collect data on all possible observables in $\sim 4\pi$ detectors

  – Ralf Gothe, Franz Klein (this AM)

- express the observables in terms of the amplitude
- fit the amplitude !

$\Downarrow$

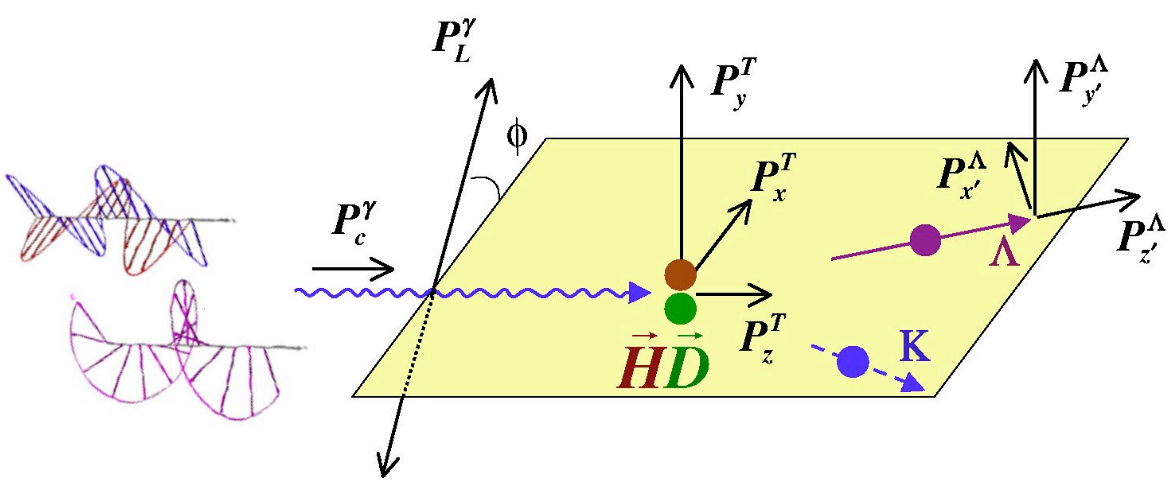
- model-independent determination of the amplitude, to within a phase

  $\Rightarrow$ I. compare/validate the total amplitude (res + bkg) in various models, (using a common ref phase)

  $\Rightarrow$ II. starting curve in the complex plane (W) for an analytic continuation to search for poles

Polarization observables in $J^{\pi} = 0^{-}$ meson photo-production :

Photon beam		Target			Recoil			Target - Recoil								
					x'	y'	z'	x'	x'	x'	y'	y'	y'	z'	z'	z'
		x	y	z				x	y	z	x	y	z	x	y	z
unpolarized	σ_0		T			P		$T_{x'}$		$L_{x'}$		Σ		$T_{z'}$		$L_{z'}$
$P_L^{\gamma}\ \sin(2\phi_{\gamma})$		H		G	$O_{x'}$		$O_{z'}$		$C_{z'}$		E		F		$-C_{x'}$	
$P_L^{\gamma}\ \cos(2\phi_{\gamma})$	$-\Sigma$		$-P$			$-T$		$-L_{z'}$		$T_{z'}$		$-\sigma_0$		$L_{x'}$		$-T_{x'}$
circular P_c^{γ}		F		$-E$	$C_{x'}$		$C_{z'}$		$-O_{z'}$		G		$-H$		$O_{x'}$	



16 different observables,
each appearing twice:

- ***single-pol observables can be measured from double-pol asy***
- ***double-pol observables can be measured from triple-pol asy***

0- N Pol Obsv table-v9 w plane

$\gamma + N \rightarrow (J^{\pi} = 0^{-}) + B^{R}(S=½)$

$\Leftrightarrow$ 4 complex amplitudes

$\rightarrow$ Cartesian CGLN (F_i) - CGLN, PR **105**(57)
$\rightarrow$ Spherical or Helicity (H_i) - BDS, NP B**95**(75)
$\rightarrow$ Transversity (b_i)

but, all are functions of θ $\leftrightarrow$ requires separate fits at each angle
$\leftrightarrow$ unknown phase angle-dependent (yuk!)

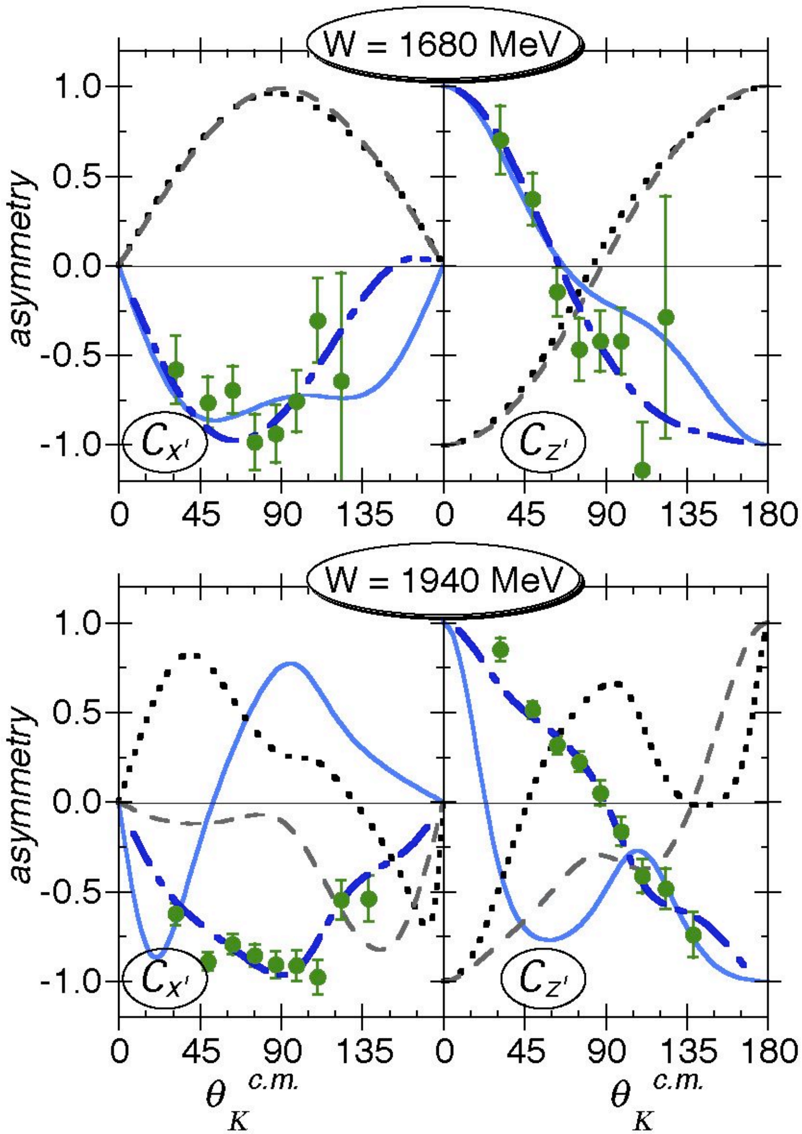
$\Rightarrow$ reduce the matrix elements to $E_{\ell\pm}$, $M_{\ell\pm}$ multipoles – independent of θ
$\rightarrow$ can use full angular distribution data in fits
$\rightarrow$ natural starting point in a search for poles

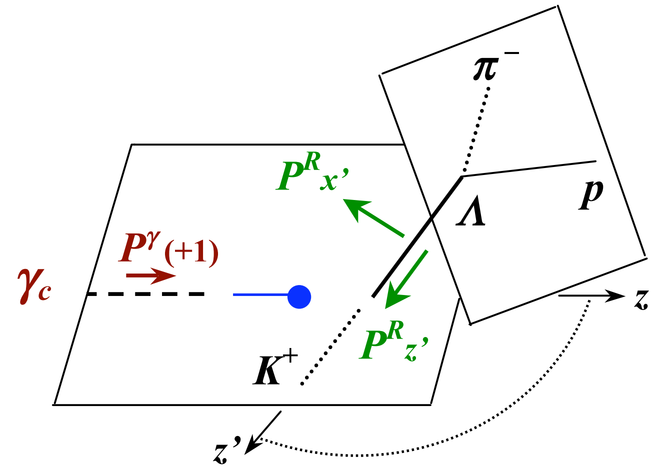
- 1st step of amplitude fitting:

write the 16 observables in terms of 4 amplitudes, and express these in terms of multipoles

- literature contains several sets of expressions;
magnitudes are identical but signs vary !

Signs of confusion in comparisons to CLAS-g1c results:





$$C_{z'} = \frac{\sigma_1^{BTR}(+1,0,+z') - \sigma_1^{BTR}(+1,0,-z')}{\sigma_1+\sigma_2}$$

⇒ **+1 at 0^0**

as in Phys Rev C**75**(07)035205

PWA

- - - - - Kaon-MAID

············ SAID

— · — · BoGa

——— JSLT PRC73 (06)

A clarifying test: construct coordinate-independent Beam-Target ratios

- specify $d\sigma^{\mathrm{B,T,R}}\left(\vec{P}^{\gamma},\,\vec{P}^{T},\,\vec{P}^{R}\right)$ in terms of $\vec{p}_{\gamma}$ (incoming photon momentum), $\vec{p}_{m}$ (outgoing meson momentum), and construct $\hat{p}_1 = \dfrac{(\vec{p}_{\gamma}\times\vec{p}_{m})\times\vec{p}_{\gamma}}{\left|(\vec{p}_{\gamma}\times\vec{p}_{m})\times\vec{p}_{\gamma}\right|}$

- construct ratios of cross sections:

$$\boldsymbol{R_E} = \frac{\left[d\sigma_1^{\mathrm{B,T,R}}\left(P_h^{\gamma}=+1,\ \vec{P}^{T}=-\hat{p}_{\gamma},\ sum\ final\right)-d\sigma_2^{\mathrm{B,T,R}}\left(P_h^{\gamma}=+1,\ \vec{P}^{T}=+\hat{p}_{\gamma},\ sum\ final\right)\right]}{\left[d\sigma_1^{\mathrm{B,T,R}}+d\sigma_2^{\mathrm{B,T,R}}\right]}$$

$$\boldsymbol{R_F} = \frac{\left[d\sigma_1^{\mathrm{B,T,R}}\left(P_h^{\gamma}=+1,\ \vec{P}^{T}=+\hat{p}_{1},\ sum\ final\right)-d\sigma_2^{\mathrm{B,T,R}}\left(P_h^{\gamma}=-1,\ \vec{P}^{T}=+\hat{p}_{1},\ sum\ final\right)\right]}{\left[d\sigma_1^{\mathrm{B,T,R}}+d\sigma_2^{\mathrm{B,T,R}}\right]}$$

$$\boldsymbol{R_G} = \frac{\left[d\sigma_1^{\mathrm{B,T,R}}\left(\phi_{\gamma}^{L}=+\pi/4,\ \vec{P}^{T}=+\hat{p}_{\gamma},\ sum\ final\right)-d\sigma_2^{\mathrm{B,T,R}}\left(\phi_{\gamma}^{L}=+\pi/4,\ \vec{P}^{T}=-\hat{p}_{\gamma},\ sum\ final\right)\right]}{\left[d\sigma_1^{\mathrm{B,T,R}}+d\sigma_2^{\mathrm{B,T,R}}\right]}$$

$$\boldsymbol{R_H} = \frac{\left[d\sigma_1^{\mathrm{B,T,R}}\left(\phi_{\gamma}^{L}=+\pi/4,\ \vec{P}^{T}=+\hat{p}_{1},\ sum\ final\right)-d\sigma_2^{\mathrm{B,T,R}}\left(\phi_{\gamma}^{L}=+\pi/4,\ \vec{P}^{T}=-\hat{p}_{1},\ sum\ final\right)\right]}{\left[d\sigma_1^{\mathrm{B,T,R}}+d\sigma_2^{\mathrm{B,T,R}}\right]}$$

Literature definitions

- BDS: Barker-Donnachie-Storrow, Nucl Phys B95 (1975)
- AS: Adelseck-Saghai, Phys Rev C42 (1990)
- FTS: Fasano-Tabakin-Saghai, Phys Rev C46 (1992)
- KDT: Knöchlein-Drechsel-Tiator, Z Phys A352 (1995)
- SHKL: Sandorfi-Hoblit-Kamano-Lee, J Phys G38 (2011)

	BDS	AS	FTS	KDT	SHKL
$\vec{P}_\gamma$	$+\hat{z}$	$-\hat{z}$	$+\hat{z}$	$+\hat{z}$	$+\hat{z}$
R_E	E	E	–E	E	E
R_F	F	–F	F	F	F
R_G	G	G	G	G	G
R_H	–H	H	H	–H	H

⇔ the same symbol/name has been used by different authors to refer to different experimental ratios;

the magnitudes are common, but the signs vary !

Resolving differences between PWA conventions with coordinate-independent ratios

- **Ratios defined with** $d\sigma^{\mathrm{B,T,R}}\left(\vec{P}^{\gamma},\, \vec{P}^{T},\, \vec{P}^{R}\right)$ **specified by** $\vec{p}_{\gamma}$ **(photon) &** $\vec{p}_{m}$ **(meson)**

- **construct** $\hat{p}_1 = \dfrac{(\vec{p}_{\gamma}\times\vec{p}_{m})\times\vec{p}_{\gamma}}{\left|(\vec{p}_{\gamma}\times\vec{p}_{m})\times\vec{p}_{\gamma}\right|}$ **,** $\vec{p}_2 = \dfrac{(\vec{p}_{\gamma}\times\vec{p}_{m})}{\left|\vec{p}_{\gamma}\times\vec{p}_{m}\right|}$ **and** $\vec{p}_3 = \dfrac{(\vec{p}_{\gamma}\times\vec{p}_{m})\times\vec{p}_{m}}{\left|(\vec{p}_{\gamma}\times\vec{p}_{m})\times\vec{p}_{m}\right|}$

- **single-pol ratios:**

$$R_S = \frac{\left[d\sigma_1^{\mathrm{B,T,R}}\left(\phi_{\gamma}^{L} = +\pi/2,\ ave\ init,\ sum\ final\right) - d\sigma_2^{\mathrm{B,T,R}}\left(\phi_{\gamma}^{L} = 0,\ ave\ init,\ sum\ final\right)\right]}{\left[d\sigma_1^{\mathrm{B,T,R}} + d\sigma_2^{\mathrm{B,T,R}}\right]}$$

$$R_T = \frac{\left[d\sigma_1^{\mathrm{B,T,R}}\left(ave\ init,\ \vec{P}^{T} = +\hat{p}_2,\ sum\ final\right) - d\sigma_2^{\mathrm{B,T,R}}\left(ave\ init,\ \vec{P}^{T} = -\hat{p}_2,\ sum\ final\right)\right]}{\left[d\sigma_1^{\mathrm{B,T,R}} + d\sigma_2^{\mathrm{B,T,R}}\right]}$$

$$R_P = \frac{\left[d\sigma_1^{\mathrm{B,T,R}}\left(ave\ init,\ ave\ init,\ \vec{P}^{R} = +\hat{p}_2\right) - d\sigma_2^{\mathrm{B,T,R}}\left(ave\ init,\ ave\ init,\ \vec{P}^{R} = -\hat{p}_2\right)\right]}{\left[d\sigma_1^{\mathrm{B,T,R}} + d\sigma_2^{\mathrm{B,T,R}}\right]}$$

- **B-T ratios:**

$$\boldsymbol{R_E} = \frac{\left[d\sigma_1^{\mathrm{B,T,R}}\left(P_h^{\gamma}=+1,\ \vec{P}^T=-\hat{p}_{\gamma},\ sum\ final\right) - d\sigma_2^{\mathrm{B,T,R}}\left(P_h^{\gamma}=+1,\ \vec{P}^T=+\hat{p}_{\gamma},\ sum\ final\right)\right]}{\left[d\sigma_1^{\mathrm{B,T,R}} + d\sigma_2^{\mathrm{B,T,R}} \right]}$$

$$\boldsymbol{R_F} = \frac{\left[d\sigma_1^{\mathrm{B,T,R}}\left(P_h^{\gamma}=+1,\ \vec{P}^T=+\hat{p}_{1},\ sum\ final\right) - d\sigma_2^{\mathrm{B,T,R}}\left(P_h^{\gamma}=-1,\ \vec{P}^T=+\hat{p}_{1},\ sum\ final\right)\right]}{\left[d\sigma_1^{\mathrm{B,T,R}} + d\sigma_2^{\mathrm{B,T,R}} \right]}$$

$$\boldsymbol{R_G} = \frac{\left[d\sigma_1^{\mathrm{B,T,R}}\left(\phi_{\gamma}^{L}=+\pi/4,\ \vec{P}^T=+\hat{p}_{\gamma},\ sum\ final\right) - d\sigma_2^{\mathrm{B,T,R}}\left(\phi_{\gamma}^{L}=+\pi/4,\ \vec{P}^T=-\hat{p}_{\gamma},\ sum\ final\right)\right]}{\left[d\sigma_1^{\mathrm{B,T,R}} + d\sigma_2^{\mathrm{B,T,R}} \right]}$$

$$\boldsymbol{R_H} = \frac{\left[d\sigma_1^{\mathrm{B,T,R}}\left(\phi_{\gamma}^{L}=+\pi/4,\ \vec{P}^T=+\hat{p}_{1},\ sum\ final\right) - d\sigma_2^{\mathrm{B,T,R}}\left(\phi_{\gamma}^{L}=+\pi/4,\ \vec{P}^T=-\hat{p}_{1},\ sum\ final\right)\right]}{\left[d\sigma_1^{\mathrm{B,T,R}} + d\sigma_2^{\mathrm{B,T,R}} \right]}$$

- **B-R ratios:**

$$\boldsymbol{R}_{C_{x'}} = \frac{\left[d\sigma_1^{\mathrm{B,T,R}}\left(P_h^{\gamma} = +1,\ ave\ init,\ \vec{P}^R = +\hat{p}_3\right) - d\sigma_2^{\mathrm{B,T,R}}\left(P_h^{\gamma} = +1,\ ave\ init,\ \vec{P}^R = -\hat{p}_3\right)\right]}{\left[d\sigma_1^{\mathrm{B,T,R}} + d\sigma_2^{\mathrm{B,T,R}} \right]}$$

$$\boldsymbol{R}_{C_{z'}} = \frac{\left[d\sigma_1^{\mathrm{B,T,R}}\left(P_h^{\gamma} = +1,\ ave\ init,\ \vec{P}^R = +\hat{p}_m\right) - d\sigma_2^{\mathrm{B,T,R}}\left(P_h^{\gamma} = +1,\ ave\ init,\ \vec{P}^R = -\hat{p}_m\right)\right]}{\left[d\sigma_1^{\mathrm{B,T,R}} + d\sigma_2^{\mathrm{B,T,R}} \right]}$$

$$\boldsymbol{R}_{O_{x'}} = \frac{\left[d\sigma_1^{\mathrm{B,T,R}}\left(\phi_{\gamma}^{L} = +\pi/4,\ ave\ init,\ \vec{P}^R = +\hat{p}_3\right) - d\sigma_2^{\mathrm{B,T,R}}\left(\phi_{\gamma}^{L} = +\pi/4,\ ave\ init,\ \vec{P}^R = -\hat{p}_3\right)\right]}{\left[d\sigma_1^{\mathrm{B,T,R}} + d\sigma_2^{\mathrm{B,T,R}} \right]}$$

$$\boldsymbol{R}_{O_{z'}} = \frac{\left[d\sigma_1^{\mathrm{B,T,R}}\left(\phi_{\gamma}^{L} = +\pi/4,\ ave\ init,\ \vec{P}^R = +\hat{p}_m\right) - d\sigma_2^{\mathrm{B,T,R}}\left(\phi_{\gamma}^{L} = +\pi/4,\ ave\ init,\ \vec{P}^R = -\hat{p}_m\right)\right]}{\left[d\sigma_1^{\mathrm{B,T,R}} + d\sigma_2^{\mathrm{B,T,R}} \right]}$$

- **T-R ratios:**

$$R_{L_{x'}} = \frac{\left[d\sigma_1^{\mathrm{B,T,R}}\left(ave\ init,\ \vec{P}^T = +\hat{p}_\gamma,\ \vec{P}^R = +\hat{p}_3\right) - d\sigma_2^{\mathrm{B,T,R}}\left(ave\ init,\ \vec{P}^T = +\hat{p}_\gamma,\ \vec{P}^R = -\hat{p}_3\right)\right]}{\left[d\sigma_1^{\mathrm{B,T,R}} + d\sigma_2^{\mathrm{B,T,R}} \right]}$$

$$R_{L_{z'}} = \frac{\left[d\sigma_1^{\mathrm{B,T,R}}\left(ave\ init,\ \vec{P}^T = +\hat{p}_\gamma,\ \vec{P}^R = +\hat{p}_m\right) - d\sigma_2^{\mathrm{B,T,R}}\left(ave\ init,\ \vec{P}^T = +\hat{p}_\gamma,\ \vec{P}^R = -\hat{p}_m\right)\right]}{\left[d\sigma_1^{\mathrm{B,T,R}} + d\sigma_2^{\mathrm{B,T,R}} \right]}$$

$$R_{T_{x'}} = \frac{\left[d\sigma_1^{\mathrm{B,T,R}}\left(ave\ init,\ \vec{P}^T = +\hat{p}_1,\ \vec{P}^R = +\hat{p}_3\right) - d\sigma_2^{\mathrm{B,T,R}}\left(ave\ init,\ \vec{P}^T = +\hat{p}_1,\ \vec{P}^R = -\hat{p}_3\right)\right]}{\left[d\sigma_1^{\mathrm{B,T,R}} + d\sigma_2^{\mathrm{B,T,R}} \right]}$$

$$R_{T_{z'}} = \frac{\left[d\sigma_1^{\mathrm{B,T,R}}\left(ave\ init,\ \vec{P}^T = +\hat{p}_1,\ \vec{P}^R = +\hat{p}_m\right) - d\sigma_2^{\mathrm{B,T,R}}\left(ave\ init,\ \vec{P}^T = +\hat{p}_1,\ \vec{P}^R = -\hat{p}_m\right)\right]}{\left[d\sigma_1^{\mathrm{B,T,R}} + d\sigma_2^{\mathrm{B,T,R}} \right]}$$

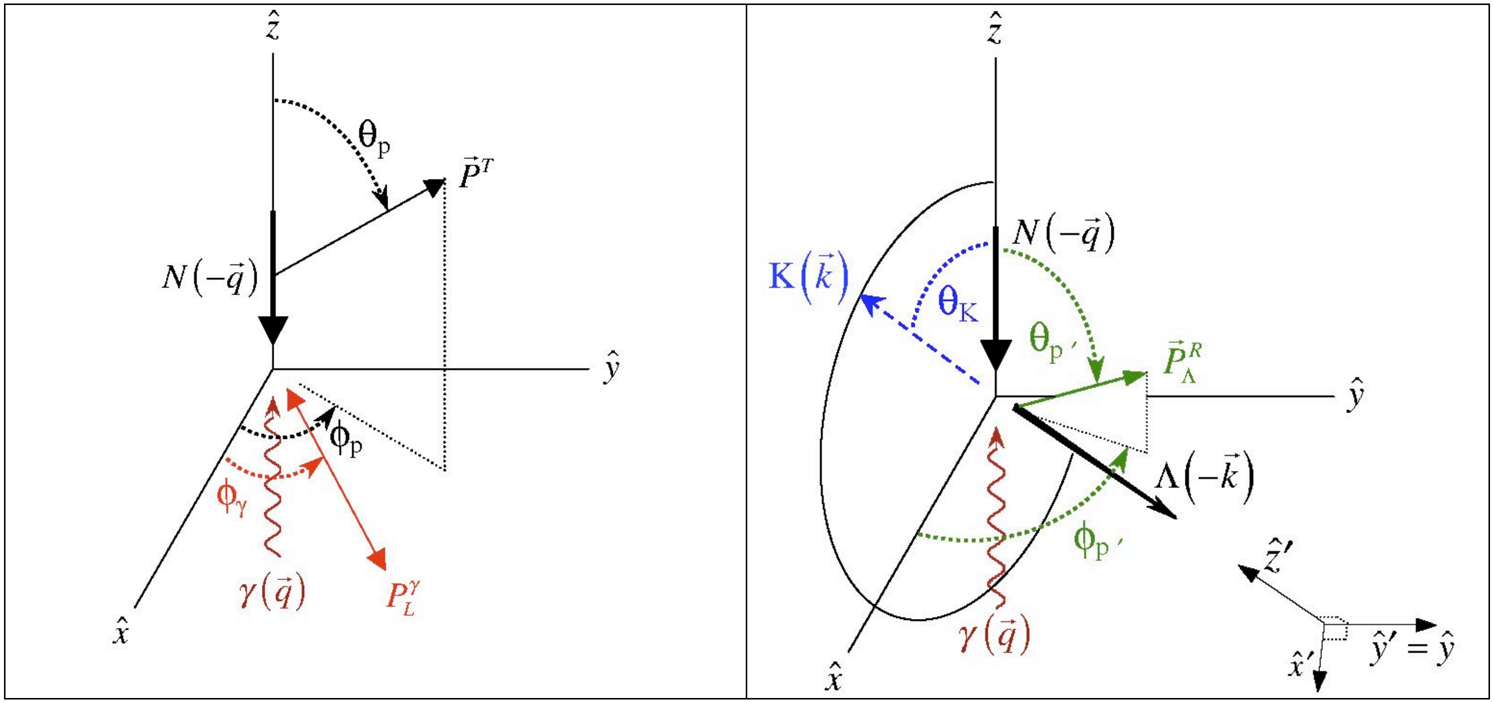
Names of experimental polarization ratios in different conventions

	MAID[1], SAID[2]	BoGa[3]	CMU[4]	SHKL[5]
R_S	Σ	Σ	Σ	Σ
R_T	T	T	T	T
R_P	P	P	P	P
R_E	E	−E	−E	E
R_F	F	F	F	F
R_G	G	G	G	G
R_H	−H	H	H	H
$R_{Cx'}$	$-C_{x'}$	$C_{x'}$	$C_{x'}$	$C_{x'}$
$R_{Cz'}$	$-C_{z'}$	$C_{z'}$	$C_{z'}$	$C_{z'}$
$R_{Ox'}$	$-O_{x'}$	$O_{x'}$	$O_{x'}$	$O_{x'}$
$R_{Oz'}$	$-O_{z'}$	$O_{z'}$	$O_{z'}$	$O_{z'}$
$R_{Lx'}$	$-L_{x'}$	$L_{x'}$	$L_{x'}$	$L_{x'}$
$R_{Lz'}$	$L_{z'}$	$L_{z'}$	$L_{z'}$	$L_{z'}$
$R_{Tx'}$	$T_{x'}$	$T_{x'}$	$T_{x'}$	$T_{x'}$
$R_{Tz'}$	$T_{z'}$	$T_{z'}$	$T_{z'}$	$T_{z'}$

[1] L. Tiator
[2] R. Workman
[3] A. Sarantsev
[4] B. Dey
[5] SHKL

new joint effort: AMS-Hoblit-Kamano-Lee, J. Phys. G38 (2011) 053001

1. explicit definitions of asymmetries in terms of angles with geometry of BDS:



2. general cross section in terms of polarizations and observables:

- derived a general analytic expression for $d\sigma(P^\gamma, P^T, P^R)$, following formalism of FTS, Fasano-Tabakin-Saghai PRC46(92), but expanding work to cover all triple polarization terms

- developed simple expressions to numerically calculate $d\sigma(P^\gamma, P^T, P^R)$, used to cross check signs.

Pseudoscalar meson photo-production

$$
\begin{aligned}
d\sigma_{(B,T,R)} = \tfrac{1}{2}\Big\{ d\sigma_0 \cdot \Big[\ 1\ & - P_L^{\gamma}\cdot P_y^{T}\cdot P_{y'}^{R}\ \cos(2\phi_\gamma)\Big] \\
+\hat{\Sigma}\cdot \Big[-P_L^{\gamma}\ \cos(2\phi_\gamma)\ & + P_y^{T}\cdot P_{y'}^{R}\ \Big] \\
+\hat{T}\cdot \Big[\ P_y^{T}\ & - P_L^{\gamma}\cdot P_{y'}^{R}\ \cos(2\phi_\gamma)\ \Big] \\
+\hat{P}\cdot \Big[\ P_{y'}^{R}\ & - P_L^{\gamma}\cdot P_y^{T}\ \cos(2\phi_\gamma)\ \Big] \\
\\
+\hat{E}\cdot \Big[-P_c^{\gamma}\cdot P_z^{T}\ & + P_L^{\gamma}\cdot P_x^{T}\cdot P_{y'}^{R}\ \sin(2\phi_\gamma)\Big] \\
+\hat{G}\cdot \Big[\ P_L^{\gamma}\cdot P_z^{T}\ \sin(2\phi_\gamma)\ & + P_c^{\gamma}\cdot P_x^{T}\cdot P_{y'}^{R}\Big] \\
+\hat{F}\cdot \Big[\ P_c^{\gamma}\cdot P_x^{T}\ & + P_L^{\gamma}\cdot P_z^{T}\cdot P_{y'}^{R}\ \sin(2\phi_\gamma)\Big] \\
+\hat{H}\cdot \Big[\ P_L^{\gamma}\cdot P_x^{T}\ \sin(2\phi_\gamma)\ & - P_c^{\gamma}\cdot P_z^{T}\cdot P_{y'}^{R}\Big] \\
\\
+\hat{C}_{x'}\cdot \Big[\ P_c^{\gamma}\cdot P_{x'}^{R}\ & - P_L^{\gamma}\cdot P_y^{T}\cdot P_{z'}^{R}\ \sin(2\phi_\gamma)\Big] \\
+\hat{C}_{z'}\cdot \Big[\ P_c^{\gamma}\cdot P_{z'}^{R}\ & + P_L^{\gamma}\cdot P_y^{T}\cdot P_{x'}^{R}\ \sin(2\phi_\gamma)\Big] \\
+\hat{O}_{x'}\cdot \Big[\ P_L^{\gamma}\cdot P_{x'}^{R}\ \sin(2\phi_\gamma)\ & + P_c^{\gamma}\cdot P_y^{T}\cdot P_{z'}^{R}\Big] \\
+\hat{O}_{z'}\cdot \Big[\ P_L^{\gamma}\cdot P_{z'}^{R}\ \sin(2\phi_\gamma)\ & - P_c^{\gamma}\cdot P_y^{T}\cdot P_{x'}^{R}\Big] \\
\\
+\hat{L}_{x'}\cdot \Big[\ P_z^{T}\cdot P_{x'}^{R}\ & + P_L^{\gamma}\cdot P_x^{T}\cdot P_{z'}^{R}\ \cos(2\phi_\gamma)\Big] \\
+\hat{L}_{z'}\cdot \Big[\ P_z^{T}\cdot P_{z'}^{R}\ & - P_L^{\gamma}\cdot P_x^{T}\cdot P_{x'}^{R}\ \cos(2\phi_\gamma)\Big] \\
+\hat{T}_{x'}\cdot \Big[\ P_x^{T}\cdot P_{x'}^{R}\ & - P_L^{\gamma}\cdot P_z^{T}\cdot P_{z'}^{R}\ \cos(2\phi_\gamma)\Big] \\
+\hat{T}_{z'}\cdot \Big[\ P_x^{T}\cdot P_{z'}^{R}\ & + P_L^{\gamma}\cdot P_z^{T}\cdot P_{x'}^{R}\ \cos(2\phi_\gamma)\Big]\ \Big\}
\end{aligned}
$$

Leading Pol <u>dependence</u>

Single Pol

beam+target

beam+recoil

target+recoil

Recoil polarization - byproduct of entrance channel angular momentum (P^γ, P^T) and the reaction physics

- recast the expression for the general cross section:

$$d\sigma^{(B,T,R)} = \tfrac{1}{2}\left[A^0 + (P^R_{x'})A^{x'} + (P^R_{y'})A^{y'} + (P^R_{z'})A^{z'} \right]$$

$$A^0 = d\sigma_0 - P_L^\gamma \cos(2\phi_\gamma)\hat{\Sigma} + P_y^T \hat{T}$$
$$- P_L^\gamma P_y^T \cos(2\phi_\gamma)\hat{P} - P_c^\gamma P_z^T \hat{E} + P_L^\gamma P_z^T \sin(2\phi_\gamma)\hat{G} + P_c^\gamma P_x^T \hat{F} + P_L^\gamma P_x^T \sin(2\phi_\gamma)\hat{H}$$

$$A^{x'} = P_c^\gamma \hat{C}_{x'} + P_L^\gamma \sin(2\phi_\gamma)\hat{O}_{x'} + P_z^T \hat{L}_{x'} + P_x^T \hat{T}_{x'}$$
$$+ P_L^\gamma P_y^T \sin(2\phi_\gamma)\hat{C}_{z'} - P_c^\gamma P_y^T \hat{O}_{z'} - P_L^\gamma P_x^T \cos(2\phi_\gamma)\hat{L}_{z'} + P_L^\gamma P_z^T \cos(2\phi_\gamma)\hat{T}_{z'}$$

$$A^{y'} = \hat{P} + P_y^T \hat{\Sigma} - P_L^\gamma \cos(2\phi_\gamma)\hat{T}$$
$$- P_L^\gamma P_y^T \cos(2\phi_\gamma) d\sigma_0 + P_L^\gamma P_x^T \sin(2\phi_\gamma)\hat{E} + P_c^\gamma P_x^T \hat{G} + P_L^\gamma P_z^T \sin(2\phi_\gamma)\hat{F} - P_c^\gamma P_z^T \hat{H}$$

$$A^{z'} = P_c^\gamma \hat{C}_{z'} + P_L^\gamma \sin(2\phi_\gamma)\hat{O}_{z'} + P_z^T \hat{L}_{z'} + P_x^T \hat{T}_{z'}$$
$$- P_L^\gamma P_y^T \sin(2\phi_\gamma)\hat{C}_{x'} + P_c^\gamma P_y^T \hat{O}_{x'} + P_L^\gamma P_x^T \cos(2\phi_\gamma)\hat{L}_{x'} - P_L^\gamma P_z^T \cos(2\phi_\gamma)\hat{T}_{x'}$$

- recoil Pol $\Rightarrow$ $(P^R_{x'}) = \dfrac{A^{x'}}{A^0}$; $(P^R_{y'}) = \dfrac{A^{y'}}{A^0}$; $(P^R_{z'}) = \dfrac{A^{z'}}{A^0}$ $\Leftrightarrow$ **SKHL, J Phys G38 (11) 053001**

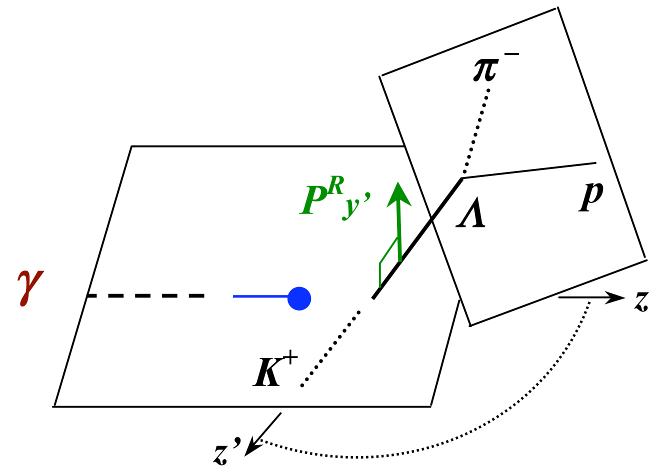
Utilizing recoil polarization

eg. 1 unpolarized beam and target:

$$A^0 = d\sigma_0$$

$$A^{x'} = 0, \quad A^{y'} = \hat{P}, \quad A^{z'} = 0$$

$$\vec{P}^R = (0, \ P, \ 0)$$

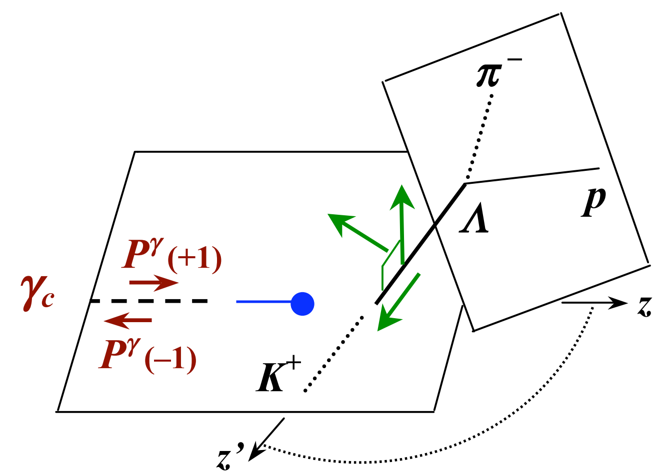


eg. 3 circularly polarized beam and unpolarized target (g1c):

$$A^0 = d\sigma_0$$

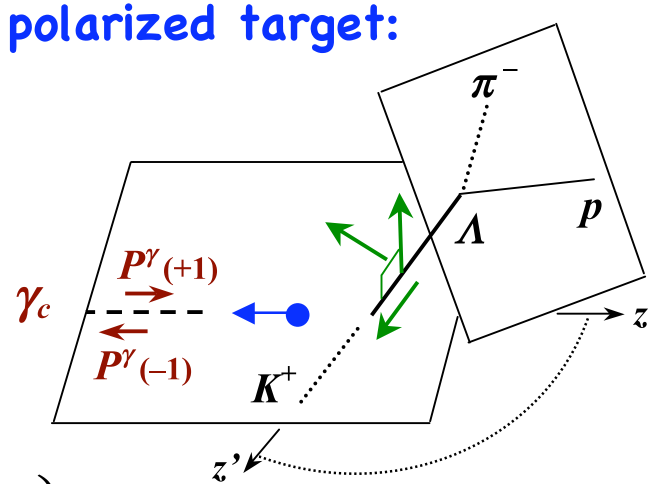
$$A^{x'} = P_c^\gamma \hat{C}_{x'}, \quad A^{y'} = \hat{P}, \quad A^{z'} = P_c^\gamma \hat{C}_{z'}$$

$$\vec{P}^R = (P_c^\gamma C_{x'}, \ P, \ P_c^\gamma C_{z'})$$



Utilizing recoil polarization

eg. 5 circularly polarized beam and longitudinally polarized target:



$$A^0 = d\sigma_0 - P_c^\gamma P_z^T \hat{E}$$

$$A^{x'} = P_c^\gamma \hat{C}_{x'} + P_z^T \hat{L}_{x'}$$

$$A^{y'} = \hat{P} - P_c^\gamma P_z^T \hat{H}$$

$$A^{z'} = P_c^\gamma \hat{C}_{z'} + P_z^T \hat{L}_{z'}$$

$$\vec{P}^R = \left(\frac{P_c^\gamma C_{x'} + P_z^T L_{x'}}{1 - P_c^\gamma P_z^T E}, \quad \frac{P - P_c^\gamma P_z^T H}{1 - P_c^\gamma P_z^T E}, \quad \frac{P_c^\gamma C_{z'} + P_z^T L_{z'}}{1 - P_c^\gamma P_z^T E} \right)$$

- sum final states (ignore recoil) : $A^0 \Rightarrow d\sigma_0$ and $E \Leftrightarrow$ P^R denominator
- average initial target pol states $(\pm P_z^T)$: $\vec{P}^R \Rightarrow C_{x'},\ P,\ C_{z'}$
- average initial beam pol states $(P_c^\gamma(h = \pm 1))$: $P_{x'}^R,\ P_{z'}^R \Rightarrow L_{x'},\ L_{z'}$
- combining initial states $P_c^\gamma \left(P_{+z}^T - P_{-z}^T \right)$: $P_{y'}^R \Rightarrow H$

$$\Updownarrow$$

nominal "transverse target asy"

⇒ **with beam and target polarized, measurements of the recoil polarization allow the extraction of all 16 observables with a single target polarization, eg. longitudinal, and thus with largely common systematics !**

- for details and other examples, see SHKL, J. Phys. G**38** (2011) 053001.

$$\sigma_0 = \left\{|F_1|^2 + |F_2|^2 + \tfrac{1}{2}\sin^2\theta \cdot \left(|F_3|^2 + |F_4|^2\right) + \Re e\left[\sin^2\theta \cdot \left(F_2^* F_3 + F_1^* F_4 + \cos\theta \cdot F_3^* F_4\right) - 2\cos\theta \cdot F_1^* F_2\right]\right\} \cdot \rho$$

$$\hat{\Sigma} = -\left[\tfrac{1}{2}\sin^2\theta \cdot \left\{|F_3|^2 + |F_4|^2\right\} + \sin^2\theta \cdot \Re e\left\{F_2^* F_3 + F_1^* F_4 + \cos\theta \cdot \left(F_3^* F_4\right)\right\}\right] \cdot \rho$$

$$\hat{T} = \Im\mathrm{m}\left\{\sin\theta\left[F_1^* F_3 - F_2^* F_4 + \cos\theta \cdot \left(F_1^* F_4 - F_2^* F_3\right) - \sin^2\theta \cdot F_3^* F_4\right]\right\} \cdot \rho$$

$$\hat{P} = \Im\mathrm{m}\left\{\sin\theta\left[-2F_1^* F_2 - F_1^* F_3 + F_2^* F_4 + \cos\theta \cdot \left(F_2^* F_3 - F_1^* F_4\right) + \sin^2\theta \cdot F_3^* F_4\right]\right\} \cdot \rho$$

$$\hat{E} = -\left[-|F_1|^2 - |F_2|^2 + \Re e\left\{2\cos\theta \cdot \left(F_1^* F_2\right) - \sin^2\theta \cdot \left(F_2^* F_3 + F_1^* F_4\right)\right\}\right] \cdot \rho$$

$$\hat{G} = +\sin^2\theta \cdot \Im m\left\{F_2^* F_3 + F_1^* F_4\right\} \cdot \rho$$

$$\hat{F} = \sin\theta \cdot \Re e\left[F_1^* F_3 - F_2^* F_4 - \cos\theta \cdot \left(F_2^* F_3 - F_1^* F_4\right)\right] \cdot \rho$$

$$\hat{H} = -\sin\theta \cdot \Im\mathrm{m}\left[2F_1^* F_2 + F_1^* F_3 - F_2^* F_4 + \cos\theta \cdot \left(F_1^* F_4 - F_2^* F_3\right)\right] \cdot \rho$$

Observables ⇔ amplitudes, CGLN F_i

- **SHKL signs**

$$\hat{O}_{x'} = -\sin\theta \cdot \Im\mathrm{m}\left[F_2^* F_3 - F_1^* F_4 + \cos\theta \cdot \left(F_2^* F_4 - F_1^* F_3\right)\right] \cdot \rho$$

$$\hat{O}_{z'} = +\sin^2\theta \cdot \Im\mathrm{m}\left[F_1^* F_3 + F_2^* F_4\right] \cdot \rho$$

$$\hat{C}_{x'} = +\sin\theta \cdot \Re e\left\{-|F_1|^2 + |F_2|^2 + F_2^* F_3 - F_1^* F_4 + \cos\theta \cdot \left(F_2^* F_4 - F_1^* F_3\right)\right\} \cdot \rho$$

$$\hat{C}_{z'} = +\Re e\left\{-2F_1^* F_2 + \cos\theta\left(|F_1|^2 + |F_2|^2\right) - \sin^2\theta \cdot \left(F_1^* F_3 + F_2^* F_4\right)\right\} \cdot \rho$$

$$\hat{L}_{x'} = +\Re e\left\{\sin\theta\left[|F_1|^2 - |F_2|^2 + \tfrac{1}{2}\sin^2\theta \cdot \left(|F_4|^2 - |F_3|^2\right) - F_2^* F_3 + F_1^* F_4 + \cos\theta\left(F_1^* F_3 - F_2^* F_4\right)\right]\right\} \cdot \rho$$

$$\hat{T}_{z'} = \Re e\left\{\sin\theta\left[-F_2^* F_3 + F_1^* F_4 + \cos\theta\left(F_1^* F_3 - F_2^* F_4\right) + \tfrac{1}{2}\sin^2\theta \cdot \left(|F_4|^2 - |F_3|^2\right)\right]\right\} \cdot \rho$$

$$\hat{L}_{z'} = \Re e\left\{2F_1^* F_2 - \cos\theta\left(|F_1|^2 + |F_2|^2\right) + \sin^2\theta \cdot \left(F_1^* F_3 + F_2^* F_4 + F_3^* F_4\right) + \tfrac{1}{2}\cos\theta\sin^2\theta \cdot \left(|F_3|^2 + |F_4|^2\right)\right\} \cdot \rho$$

$$\hat{T}_{x'} = \Re e\left\{\sin^2\theta\left[-F_1^* F_3 - F_2^* F_4 - F_3^* F_4 - \tfrac{1}{2}\cos\theta \cdot \left(|F_3|^2 + |F_4|^2\right)\right]\right\} \cdot \rho$$

Fierz identities relating asymmetries: Chiang & Tabakin, PRC**55**(97) –with SHKL signs

(L.0) $$1=\frac{1}{3}\left[\ \Sigma^2+T^2+P^2+E^2+G^2+F^2+H^2+O_{x'}^2+O_{z'}^2+C_{x'}^2+C_{z'}^2\ \ +L_{x'}^2+L_{z'}^2+T_{x'}^2+T_{z'}^2\ \right]$$

(L.TR)	$\Sigma=+T\,P+T_{x'}L_{z'}-T_{z'}L_{x'}$	(Q.b)	$C_{x'}O_{x'}+C_{z'}O_{z'}+E\,G-F\,H=0$
(L.BR)	$T=+\Sigma P-C_{x'}O_{z'}+C_{z'}O_{x'}$	(Q.t)	$G\,H-E\,F-L_{x'}T_{x'}-L_{z'}T_{z'}=0$
(L.BT)	$P=+\Sigma T+G\,F+E\,H$	(Q.r)	$C_{x'}C_{z'}+O_{x'}O_{z'}-L_{x'}L_{z'}-T_{x'}T_{z'}=0$
(L.1)	$G=+P\,F+O_{x'}L_{x'}+O_{z'}L_{z'}$	(Q.bt.1)	$\Sigma G-T\,F-O_{z'}T_{x'}+O_{x'}T_{z'}=0$
(L.2)	$H=+P\,E+O_{x'}T_{x'}+O_{z'}T_{z'}$	(Q.bt.2)	$\Sigma H-T\,E+O_{z'}L_{x'}-O_{x'}L_{z'}=0$
(L.3)	$E=+P\,H-C_{x'}L_{x'}-C_{z'}L_{z'}$	(Q.bt.3)	$\Sigma E-T\,H+C_{z'}T_{x'}-C_{x'}T_{z'}=0$
(L.4)	$F=+P\,G+C_{x'}T_{x'}+C_{z'}T_{z'}$	(Q.bt.4)	$\Sigma F-T\,G+C_{z'}L_{x'}-C_{x'}L_{z'}=0$
(L.5)	$O_{x'}=+T\,C_{z'}+G\,L_{x'}+H\,T_{x'}$	(Q.br.1)	$\Sigma O_{x'}-P\,C_{z'}+G\,T_{z'}-H\,L_{z'}=0$
(L.6)	$O_{z'}=-T\,C_{x'}+G\,L_{z'}+H\,T_{z'}$	(Q.br.2)	$\Sigma O_{z'}+P\,C_{x'}-G\,T_{x'}+H\,L_{x'}=0$
(L.7)	$C_{x'}=-T\,O_{z'}-E\,L_{x'}+F\,T_{x'}$	(Q.br.3)	$\Sigma C_{x'}+P\,O_{z'}-E\,T_{z'}-F\,L_{z'}=0$
(L.8)	$C_{z'}=+T\,O_{x'}-E\,L_{z'}+F\,T_{z'}$	(Q.br.4)	$\Sigma C_{z'}-P\,O_{x'}+E\,T_{x'}+F\,L_{x'}=0$
(L.9)	$T_{x'}=+\Sigma L_{z'}+H\,O_{x'}+F\,C_{x'}$	(Q.tr.1)	$T\,T_{x'}-P\,L_{z'}-H\,C_{z'}+F\,O_{z'}=0$
(L.10)	$T_{z'}=-\Sigma L_{x'}+H\,O_{z'}+F\,C_{z'}$	(Q.tr.2)	$T\,T_{z'}+P\,L_{x'}+H\,C_{x'}-F\,O_{x'}=0$
(L.11)	$L_{x'}=-\Sigma T_{z'}+G\,O_{x'}-E\,C_{x'}$	(Q.tr.3)	$T\,L_{x'}+P\,T_{z'}-G\,C_{z'}-E\,O_{z'}=0$
(L.12)	$L_{z'}=+\Sigma T_{x'}+G\,O_{z'}-E\,C_{z'}$	(Q.tr.4)	$T\,L_{z'}-P\,T_{x'}+G\,C_{x'}+E\,O_{x'}=0$

(S.bt)	$G^2+H^2+E^2+F^2+\Sigma^2+T^2-P^2=1$	(S.b)	$G^2+H^2-E^2-F^2-O_{x'}^2-O_{z'}^2+C_{x'}^2+C_{z'}^2=0$
(S.br)	$O_{x'}^2+O_{z'}^2+C_{x'}^2+C_{z'}^2+\Sigma^2-T^2+P^2=1$	(S.t)	$G^2-H^2+E^2-F^2+T_{x'}^2+T_{z'}^2-L_{x'}^2-L_{z'}^2=0$
(S.tr)	$T_{x'}^2+T_{z'}^2+L_{x'}^2+L_{z'}^2-\Sigma^2+T^2+P^2=1$	(S.r)	$O_{x'}^2-O_{z'}^2+C_{x'}^2-C_{z'}^2-T_{x'}^2+T_{z'}^2-L_{x'}^2+L_{z'}^2=0$